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PENETRATION OF HIGH EXPLOSIVES BY INERT PROJECTILES

FINAL REPORT

M. H. RICE

NOVEMBER, 1977

U. S. ARMY RESEARCH OFFICE

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SYSTEMS, SCIENCE AND SOFTWARE
P. O. BOX 1620
LA JOLLA, CALIFORNIA 92038

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A series of two-dimensional calculations has been carried out for the penetration of the high explosives PBX-9404 and Composition B by steel spheres. Equations-of-state and reaction-rate models are presented which, when used in the two-dimensional HELP code, result in good agreement with the observed velocity-loss data and detonation thresholds.

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TABLE OF CONTENTS

I. INTRODUCTION

The two-dimensional, Eulerian computer code HELP
(Hydrodynamics ELastic Plastic) developed at Systems, Science
and Software (S³) has been modified to allow for the inclusion
of an explicit reaction rate for modeling partially reacted
states of high explosives. Calculations of the detonation
thresholds and velocity losses of steel spheres penetrating
slabs of Composition B and PBX-9404 have been carried out.
Empirical reaction-rate models based on experimental, one-
dimensional run-to-detonation data (Reference 1) give good
correlations with the experimental thresholds and velocity-
loss data as obtained at the Ballistic Research Laboratories
(BRL).

The new capability provided by the code should be of
use in several problem areas of interest to the Army. In
particular, work is already underway to determine the effect
of explosive reaction rates on shaped-charge jet formation.

Section II of this report contains a description of
the reaction-rate models and the correlations of calculated
and experimental velocity-loss and detonation threshold
data. The equation of state and constitutive models used in
the calculations are described in Section III. The reaction-
rate models and their experimental basis are discussed in
Section IV.

II. HELP CODE CALCULATIONS

The two-dimensional, Eulerian HELP code (Reference 2) has been employed extensively in a variety of studies to calculate the material flow associated with both the impact process and the detonation of high explosives as well as the interaction between detonated products and inert material. However, prior to the present work, the explosive computational cell was treated as either completely unreacted or completely reacted. In addition, it had to be decided before hand whether a given cell was to detonate, and if so, at what time. The modification to allow for the use of an explicit reaction rate enables one to treat partially reacted states of the HE. Correlations can be made of the degree of decomposition with the impact velocity of a projectile, the initial projectile/target configuration and the material properties of both the HE and the projectile. In particular, the use of a realistic rate law enables one to determine whether a given impact and configuration will lead to only partial decomposition or to complete detonation.

2.1 CORRELATIONS WITH EXPERIMENT

In the present work the modified HELP code was used for a series of calculations consisting of a 1.3 cm diameter steel sphere striking at normal incidence a 10 cm diameter, 2.5 cm thick cylindrical slab of Composition B or PBX-9404. The initial calculational configuration is shown in Figure 1. The quantities of particular interest are the velocity loss of the projectile, $V_\ell \equiv V_s - V_f$, as a function of the initial striking velocity, V_s, and the minimum striking velocity required to produce complete detonation of the slab. A comparison of calculated velocity losses with experimental data as supplied by the BRL (Reference 3) is given in Figure 2. Several of the calculations, as indicated in the figure, were
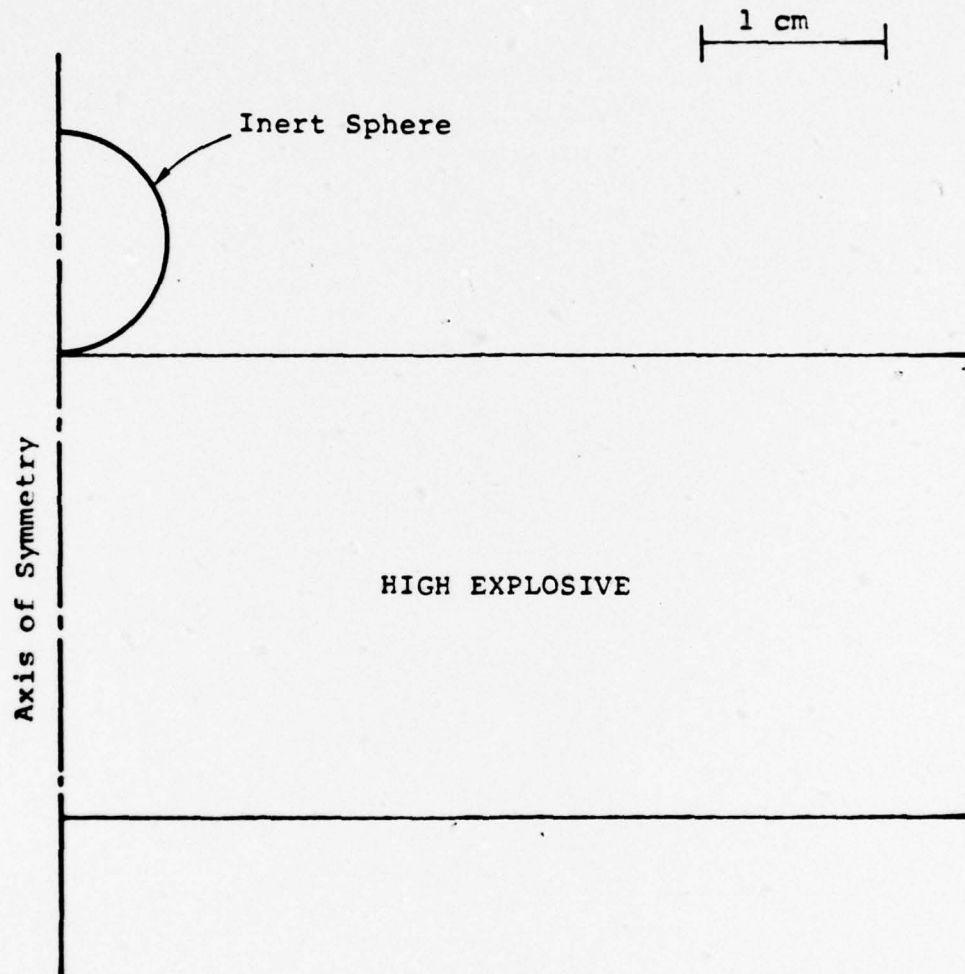
Figure 1. Initial configuration for the numerical calcula-
tions involving inert spheres perforating high
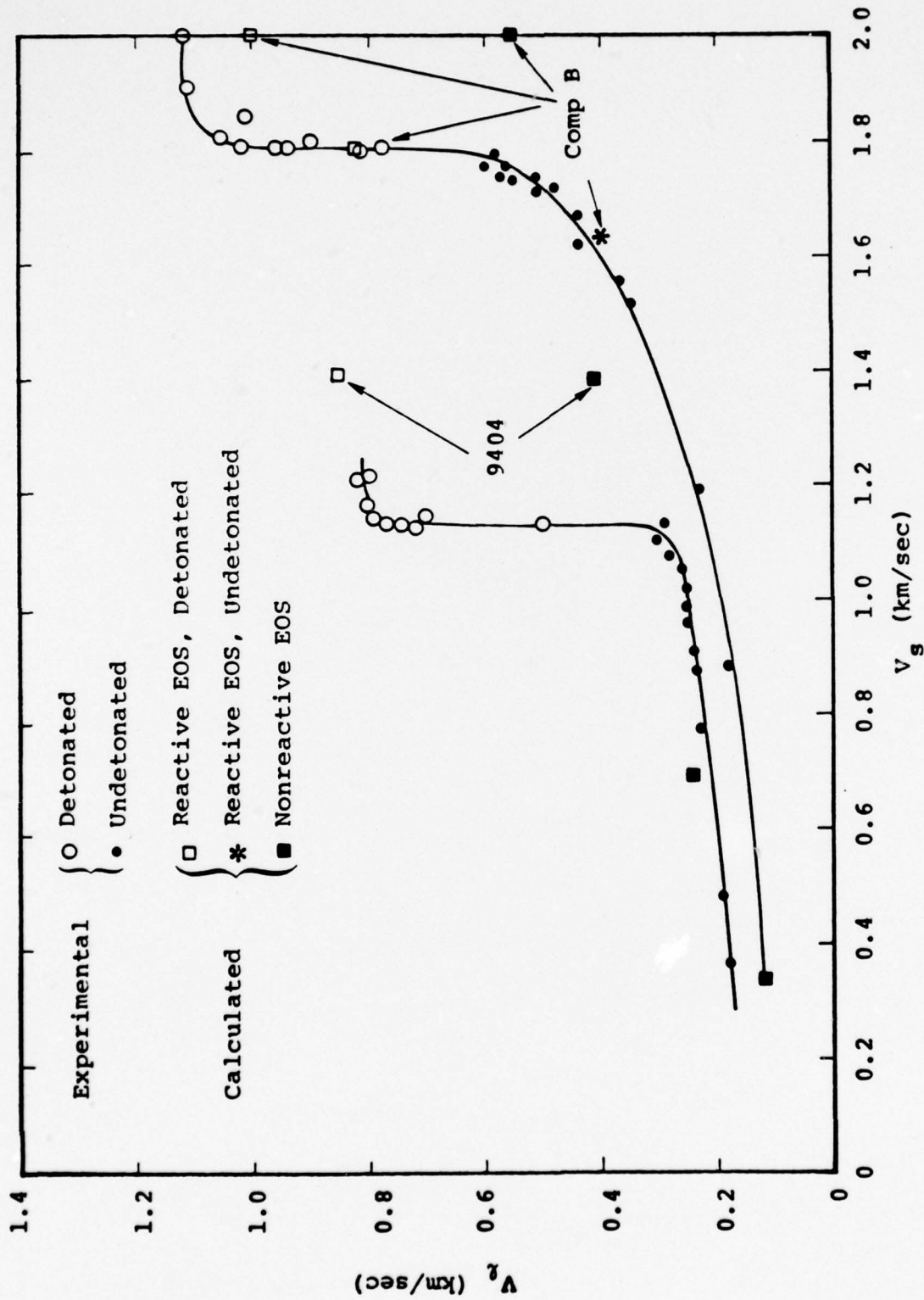explosive targets.

Figure 2. Velocity loss versus striking velocity for a 1.3 cm diameter steel sphere impacting a 10 cm diameter, 2.5 cm thick cylinder of high explosive.

performed using the completely unreacted equation of state
for the HE, as given in Section 3.2. The agreement of these
points with the experimental curves is completely satisfactory.
Four of the points in Figure 2 are the result of calculations
using the reactive equation of state for HE (Section 3.2) and
the reaction rate models as given in Figure 3, which is a plot
of the fractional rate of decomposition as a function of pres-
sure. As can be seen from Figure 2, the results of these cal-
culations are also in good agreement with the experimental
results.

A summary of the results of the calculations for PBX-
9404 is given in Table I. The reaction rates used in these
calculations are those as given by the "Forest Fire" model,
proposed by Mader and Forest (Reference 1). A brief descrip-
tion of the model is given in Section 4.1. The reaction
rates are obtained from experimental, one-dimensional run-to-
detonation data and are a function of a single equation of
state variable which may be taken either as the pressure or
the temperature. In the present calculation pressure was
used as the independent variable. The Forest Fire reaction
rates for PBX-9404 are plotted as a function of pressure in
Figure 3. The use of this model in the HELP code calculations
results in a transition from partial to complete reaction for
striking velocities in the range 1.0 to 1.1 km/sec, in good
agreement with the experimental results.

A summary of the results of calculations for Composition
B is given in Table II. For Composition B, the use of the
Forest Fire reaction rates results in only partial decomposi-
tion at a striking velocity of 2.0 km/sec. Increasing the
reaction rates by a factor of 1.5 at each pressure brings
the calculated detonation threshold to between 1.7 and 1.8
km/sec, in agreement with the experimental results. These
modified reaction rates for Composition B are plotted in
Figure 3.

Reaction Rate ($-\frac{1}{w}\frac{dw}{dt}$ µsec^{-1})

1000

100

10

1.0

0.1

0.01

PBX-9404

Composition B

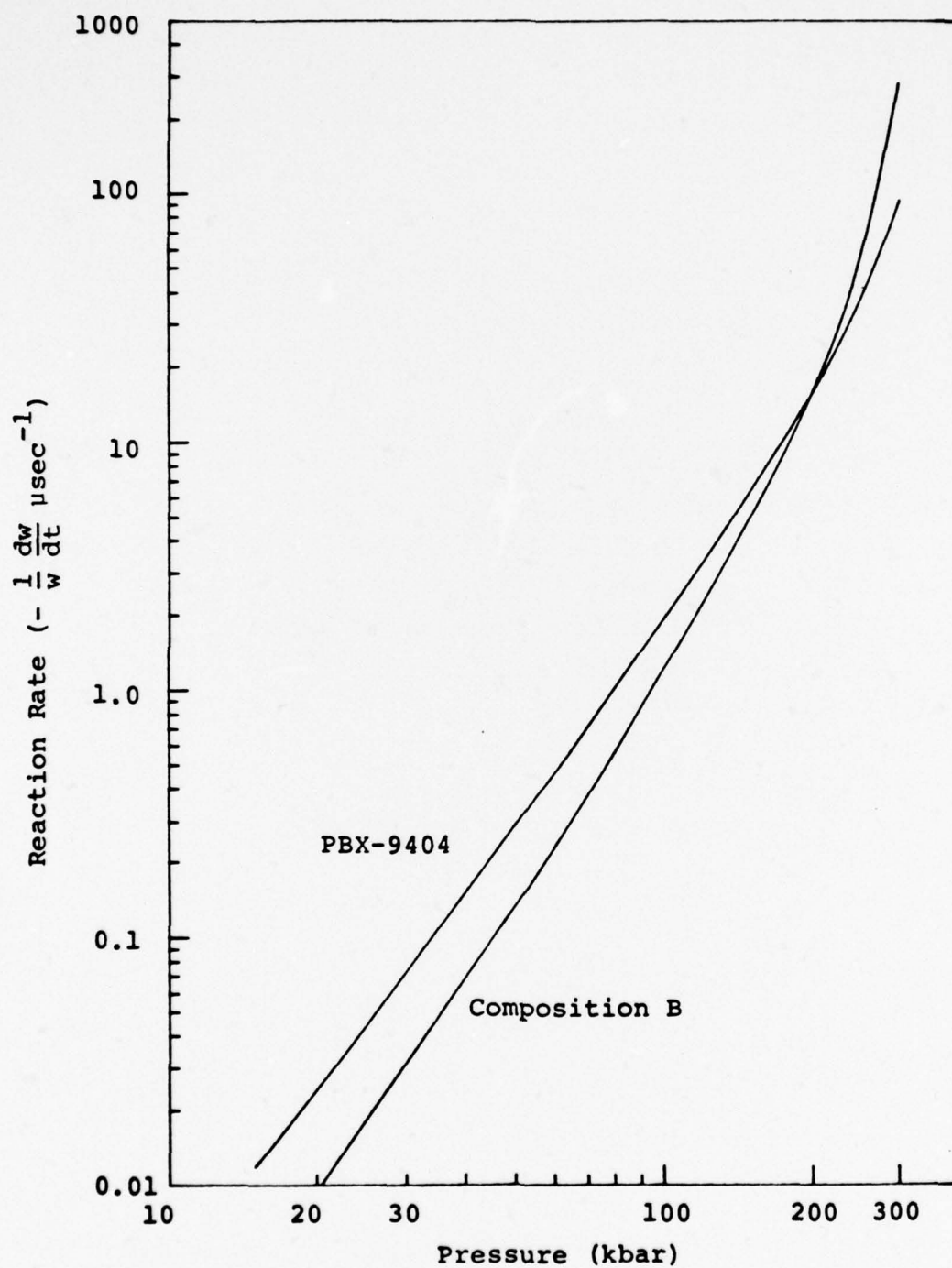10   20   30   100   200   300

Pressure (kbar)

Figure 3.   Reaction rate versus pressure as used in the HELP
code calculations, where w is the mass fraction of
unreacted explosive.

TABLE I

HELP CODE RESULTS FOR THE PENETRATION OF PBX-9404
BY A 1.3 CM DIAMETER STEEL SPHERE

REACTION RATE MODEL	INITIAL VELOCITY (km/sec)	VELOCITY LOSS (km/sec)	DETONATION?
Inert	0.35	0.125	--
Inert	0.7	0.23	--
Inert	1.4	0.41	--
Forest Fire	0.9	--	No
Forest Fire	1.0	--	No
Forest Fire	1.1	--	Yes
Forest Fire	1.4	0.86	Yes

TABLE II

HELP CODE RESULTS FOR THE PENETRATION OF COMPOSITION B
BY A 1.3 CM DIAMETER STEEL SPHERE

REACTION RATE MODEL	INITIAL VELOCITY (km/sec)	VELOCITY LOSS (km/sec)	DETONATION?
Inert	2.0	0.56	--
Forest Fire	2.0	--	No
FF × 1.5	2.0	1.0	Yes
FF × 1.5	1.8	0.85	Yes
FF × 1.5	1.75	--	Yes
FF × 1.5	1.70	--	No
FF × 1.5	1.6	0.39	No
FF × 1.5 $Y_s × 10$	1.6	0.28*	No
FF × 1.5 $Y_s = 0$	1.6	0.28*	No

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*Velocity loss at 10 µsec

10

In summary, the equations of state and constitutive models described in Section III together with the Forest Fire reaction rate model give good agreement between calculated velocity-loss and detonation threshold data for PBX-9404. For Composition B it is necessary to increase the Forest Fire reaction rates by a factor of 1.5 in order to obtain agreement between the calculated and experimental detonation thresholds. With regard to the factor of 1.5, since the Forest Fire reaction rates were obtained with the HOM equation of state for partially reacted explosive (Reference 4) instead of the models used in the HELP code calculations, a study was undertaken to determine the sensitivity of the reaction rates to the particular equation of state models used in their calculation. The results of this investigation are given in Section 4.2.

2.2   SENSITIVITY TO UNREACTED EXPLOSIVE YIELD STRENGTH

Two calculations for Composition B were performed in order to determine the sensitivity of the calculated detonation threshold to the value of the yield strength used for the unreacted explosive. For one calculation the yield strength was increased by a factor of 10 over the nominal value of 0.512 kbar (Section 3.2.1), and for the other calculation the yield strength was reduced to zero. There was no discernible effect on the detonation threshold and a negligible, if any, effect on the calculated velocity loss. It should be emphasized that this insensitivity to the value of the yield stress is for an underlined unconfined explosive geometry at a rather high striking velocity. For a cased configuration at lower impact velocities, the value of the yield strength and its effect on the amount of plastic deformation would probably be significant.

III. MATERIAL MODELS

3.1   INERT MATERIAL MODEL AND PARAMETERS FOR STEEL

The inert material model employed in HELP includes an equation of state, a deviatoric constitutive relation for elastic and plastic deformations, and a failure criterion, as discussed in the following paragraphs.

For steel, the Tillotson (Reference 5) equation of state, modified to give a smooth transition between compressed and expanded states was used. For compressed states, i.e., when $\rho/\rho_0 > 1$, or for any cold states, $E < E_s$, where E_s is the specific internal energy at incipient vaporization, the equation of state has the form

$$P = P_c = \left[a + \frac{b}{\frac{E}{E_0 \xi^2} + 1} \right] E\rho + A\eta + B\eta^2 \tag{3.1}$$

where $\xi \equiv \rho/\rho_0$ and $\eta \equiv \xi - 1$. For expanded hot states, i.e., when $\rho/\rho_0 < 1$ and $E > E_s'$, the equation of state has the form

$$P = P_E = aE\rho + \left[\frac{bE\rho}{\frac{E}{E_0 \xi^2} + 1} + A\eta e^{-\beta \left(\frac{V}{V_0} - 1 \right)} \right]$$

$$\times e^{-\alpha \left(\frac{V}{V_0} - 1 \right)^2} . \tag{3.2}$$

A smooth transition between the compressed and expanded states is insured by a transition equation for the intermediate region defined by $E_s < E < E_s'$ and $\rho/\rho_0 < 1$, where E_s' is the specific internal energy at complete vaporization. This blended portion of the equation of state has the form

12

$$P = \frac{(E-E_s)P_E + (E'_s-E)P_C}{(E'_s - E_s)} .$$
(3.3)

The values of the parameters as used for steel in the above equations are

A = 1280 kbar

B = 650 kbar

a = 0.5

b = 1.5

$E_0 = 9.5 \times 10^{10}$ erg/g

$E_s = 2.44 \times 10^{10}$ erg/g

$E'_s = 10.2 \times 10^{10}$ erg/g

α = 5.0

β = 5.0

The deviatoric stress increments, dS'_{ij} are calculated from the elastic relation

$$dS'_{ij} = 2\mu d\varepsilon'_{ij},$$
(3.4)

where μ is the shear modulus and $d\varepsilon'_{ij}$ are the increments of deviatoric strain. When such an increment of stress causes the von Mises yield criterion,

$$S'_{ij} S'_{ij} \leq 2Y_s^2$$

to be violated, each stress component is proportionately reduced to bring the stress back (normally) to the yield surface. A variable yield strength

13

$$Y_s = \left(Y_0 + Y_1 \eta + Y_2 \eta^2 \right) \left(1 - \frac{E}{E_m} \right) \tag{3.5}$$

can be used to account for the increase in strength at high pressures and the decrease in strength at elevated values of the specific internal energy, where E_m is the internal energy at the melt point. The values of the parameters as used for steel are

μ = 800 kbar

Y_0 = 6.0 kbar

Y_1 = 0

Y_2 = 0

E_m = 1.3 × 10^{10} erg/g

The failure criterion employed in the HELP code is based upon a minimum allowable compression ratio, η_{min}. When the material in a computational cell is subjected to conditions such that the failure criterion is violated, i.e.,

$\eta < \eta_{min}$,

the material is considered failed, and the stresses in that cell are set to zero. For steel, the value used for the minimum compression ratio was

η_{min} = -0.02

14

3.2   HIGH-EXPLOSIVE EQUATION OF STATE

Partially reacted explosive is treated as a mixture of completely reacted and completely unreacted components in pressure equilibrium. Separate equations of state of the general form

$$P_1 = f_1 \ (\rho_1, \ E_1)$$

$$P_2 = f_2 \ (\rho_2, \ E_2) \tag{3.6}$$

are used for the unreacted and reacted components, respectively, where ρ_1, ρ_2 and E_1, E_2 are the densities and specific internal energies. The equations of state are described in detail in the following sections.

The pressure is calculated as follows: at the end of a computational cycle the overall density ρ and the specific internal energies of the unreacted and reacted components are known. The general expression relating the overall density to the individual densities of the unreacted and reacted components is

$$\frac{1}{\rho} = \frac{w}{\rho_1} + \frac{(1-w)}{\rho_2} \ , \tag{3.7}$$

where w is the mass fraction of unreacted explosive in the computational cell. The individual densities are varied in an iterative procedure, subject to the above constraint, until the calculated pressures of the unreacted and reacted components are equal. Thus, the determination of the pressure in a cell containing partially reacted high-explosive is formally the same as that for a cell containing a mixture of any other two material constituents, and the computational machinery developed previously for use in the HELP code was immediately applicable.

15

The determination of the mass fraction of unreacted explosive in a computational cell is a two-step procedure. The first possible variation is due to the mass flow into and out of a given cell from neighboring cells with different unreacted mass fractions. This increment is calculated at the same time that the cell mass is updated, a mass-weighted average of the appropriate contributions being used. The second variation is explicit change as calculated from the assumed reaction rate of the explosive decomposition. A separate subroutine was written to perform this part of the update.

3.2.1 Unreacted Composition B

For compressed states, $\rho > \rho_0$, a Mie-Gruneisen equation of state was used,

$$P_c = f(\rho) + \rho GE, \tag{3.8}$$

where

$$f(\rho) = \left(1 - \frac{G}{2} \xi\epsilon\right) P_H(\rho) \tag{3.9}$$

in terms of the Hugoniot pressure $P_H(\rho)$, and

$$\xi \equiv \frac{\rho}{\rho} , \quad \epsilon \equiv 1 - \frac{V}{V_0} . \tag{3.10}$$

The Hugoniot pressure is determined from the straight line U_s, U_p fit,

$$U_s = C_0 + SU_p , \tag{3.11}$$

16

or equivalently,

$$P_H = \rho_0 C_0^2 \, \epsilon / (1 - S\epsilon)^2 \, . \tag{3.12}$$

For negative pressures the Hugoniot was extrapolated as a straight line. Values of the parameters in the above equations as obtained from Reference 1 are

$$\rho_0 = 1.715 \text{ g/cm}^3$$
$$C_0 = 2.31 \times 10^5 \text{ cm/sec}$$
$$S = 1.83$$
$$G = 1.5 \, .$$

Other parameters as obtained from the Ballistic Research Laboratories (Reference 6) are the shear modulus,

$$\mu = 50.3 \text{ kbar}$$

the yield stress in simple tension of

$$Y_T = 0.887 \text{ kbar}$$

from which the von Mises yield criterion is

$$Y_s = Y_T/\sqrt{3} = 0.512 \text{ kbar,}$$

and the tensile strength of

$$-S^* = 0.1042 \text{ kbar.}$$

The volumetric strain at failure as used in the HELP code is

then

$$\epsilon_{min} = S^*/(\rho_0 C_0^2) = -0.00114$$

For expanded hot states the equation of state used was

$$P_E = \rho GE + \rho_0 C_0^2 \left(\frac{\rho}{\rho_0} - 1\right) \exp\left[-\beta\left(\frac{V}{V_0} - 1\right)\right]$$

$$\times \exp\left[-\alpha\left(\frac{V}{V_0} - 1\right)^2\right] . \tag{3.13}$$

For intermediate states, the blend as described in Section 3.1 was used. For Composition B, the energy at the melt point was taken as

$$E_m = 8.29 \times 10^8 \text{ erg/g}$$

and the energy at incipient vaporization and complete vaporization were estimated as

$$E_s = 9.19 \times 10^9 \text{ erg/g}$$

$$E_s' = 3.0 \times 10^{10} \text{ erg/g}$$

The standard values $\alpha = \beta = 5.0$ were used in Equation (3.13).

3.2.2 Unreacted PBX-9404

The Mie-Gruneisen equation of state, as described in the preceding section, was also used for compressed states

of PBX-9404. Values of the parameters as obtained from Reference 7 are

$$\rho_0 = 1.83 \text{ g/cm}^3$$

$$C_0 = 2.423 \times 10^5 \text{ cm/sec}$$

$$S = 1.883$$

$$G = 0.675$$

The shear modulus, as calculated from the shear wave velocity of 1.62×10^5 cm/sec (Reference 8) is

$$\mu = \rho_0 C_t^2 = 48.0 \text{ kbar.}$$

The form of the expanded phase equation of state was taken to be the same as that for Composition B, Equation (3.13), with $\alpha = \beta = 5.0$. The melt temperature was taken as 280°C, which together with specific heat data from Reference 9 gives for the energy at melt,

$$E_m = 3.21 \times 10^9 \text{ erg/g.}$$

The energies for incipient vaporization and complete vaporization were estimated as

$$E_s = 1.16 \times 10^{10} \text{ erg/g}$$

$$E_s' = 3.25 \times 10^{10} \text{ erg/g .}$$

19

The yield and failure parameters were taken to be the same as those for Composition B, i.e., the shear yield stress for use in the von Mises yield criterion is

$$Y_s = 0.512 \text{ kbar},$$

and the tensile strength as

$$-S^* = 0.1042 \text{ kbar}.$$

The resulting volumetric strain at failure is then

$$\varepsilon_{min} = S^*/(\rho_0 C_0^2) = -0.00097$$

3.2.3 Reacted Composition B

The Jones-Wilkins-Lee (JWL) equation of state was used for both Composition B and PBX-9404 for the completely reacted state. It had already been coded for use in the HELP code and is of the form

$$P = A \left(1 - \frac{\omega}{R_1 V}\right) \exp\,(-R_1 V) + B \left(1 - \frac{\omega}{R_2 V}\right) \exp\,(-R_2 V)$$

$$+ \frac{\omega E}{V}\,, \tag{3.14}$$

where V is the volume of the detonation products relative to that of the undetonated explosive, and E is the internal energy per cm^3 of undetonated explosive.

The values of the parameters for Composition B as taken from Reference 10 are

$$A = 5.242 \text{ Mbar}$$

20

B = 0.07678 Mbar

R_1 = 4.20

R_2 = 1.10

ω = 0.34

E_0 = 0.085 Mbar-cc/cc

3.2.4 Reacted PBX-9404

The parameters for the JWL equation of state as obtained from Reference 10 are

A = 8.545 Mbar

B = 0.2049 Mbar

R_1 = 4.60

R_2 = 1.35

ω = 0.25

E_0 = 0.102 Mbar-cc/cc

IV.   REACTION-RATE MODEL

4.1   FOREST FIRE REACTION-RATE MODEL

A brief description of the model is given here; a detailed discussion is given by Mader and Forest (Reference 1).

The basic assumption is that the build-up to complete detonation takes place along a unique curve in distance, time and state space, regardless of the initial driving pressure. The shock pressure as a function of distance run is then given by the so-called "Pop plot", which is a fit to the experimental data for initial shock pressure as a function of run to detonation. The Pop plot for Composition B is shown in Figure 4 along with the assumed growth of the shock pressure as complete detonation is approached. A second assumption, as indicated in Figure 4, is that the pressure gradient immediately behind the shock front is zero.

The additional information required for the determination of the reaction rates are the Hugoniot curve for shocks to the intermediate, partially reacted states, and an equation of state of the form

$$P = f\ (\rho, E, w),\tag{4.1}$$

where ρ is the density, E the specific internal energy, w the mass fraction of unreacted explosive. The calculational procedure is as follows:

For a given shock pressure P_H, the experimental curve of U_s versus U_p for the partially reacted Hugoniot,

$$U_s = C + S'U_p,\tag{4.2}$$

and the Hugoniot relations are used to determine the state at the shock front,

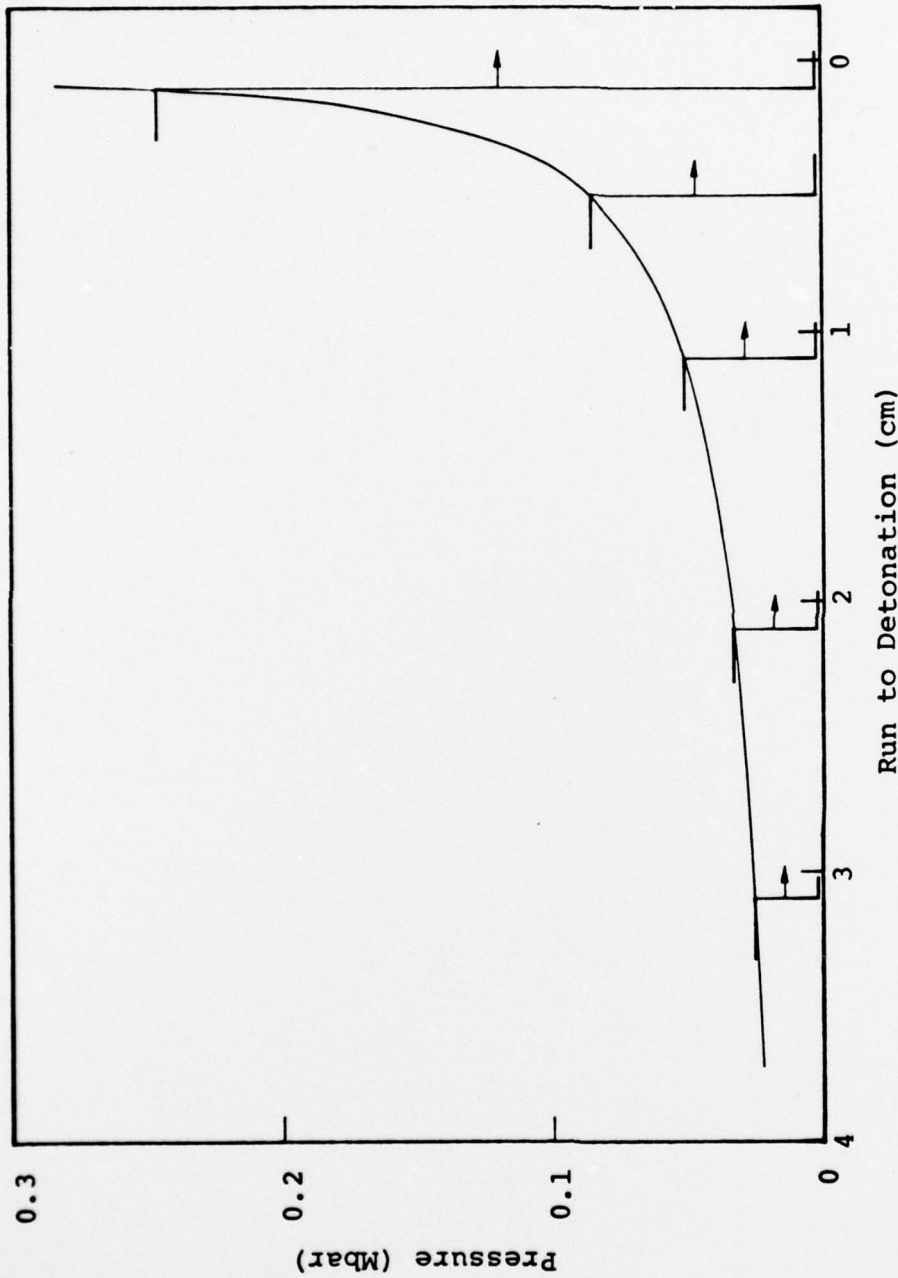$$P_H = \rho_0 U_s U_p = \rho_0\ (C + S'\ U_p)U_p\tag{4.3}$$

Figure 4. Run to detonation versus initial driving pressure ("Pop Plot") for Composition B. The figure also shows the assumed growth of a pressure pulse as complete detonation is approached.

$$V_H = V_0 \ (U_s - U_p)/U_s \tag{4.4}$$

$$E_H = \frac{1}{2} \ U_p^2 \ . \tag{4.5}$$

These values are used with the equation of state to determine the mass fraction of unreacted explosive at the shock front,

$$P_H = f \ (\rho_H, \ E_H, \ w_H). \tag{4.6}$$

The reaction rate itself is then obtained from the Lagrangian time derivative of the above expression,

$$\left(\frac{\partial P}{\partial t}\right)_m = f_1 \ \left(\frac{\partial \rho}{\partial t}\right)_m + f_2 \ \left(\frac{\partial E}{\partial t}\right)_m + f_3 \ \left(\frac{\partial w}{\partial t}\right)_m \ , \tag{4.7}$$

where the subscript m denotes the time derivative at a fixed mass element, and $f_{1,2,3}$ are the partial derivatives of the equation of state function with respect to its arguments.

At this point, the assumption of a zero pressure gradient at the shock front is used to evaluate the partial derivatives. The general expression

$$\left(\frac{\partial P}{\partial t}\right)_m \equiv \left(\frac{\partial P}{\partial t}\right)_x + \left(\frac{\partial P}{\partial x}\right)_t \ \left(\frac{\partial x}{\partial t}\right)_m$$

reduces to

$$\left(\frac{\partial P}{\partial t}\right)_m = \left(\frac{\partial P}{\partial t}\right)_x = \frac{dP_H}{dt} = - \ \frac{dP_H}{dR} \ U_s \tag{4.8}$$

24

where the derivatives of the Hugoniot pressure with respect to R (run to detonation) is obtained from the Pop plot. The Lagrangian equation for conservation of momentum

$$\left(\frac{\partial u}{\partial t}\right)_m = - \left(\frac{\partial P}{\partial m}\right)_t = 0$$

allows one to write for the rate of change of particle velocity at the shock front

$$\frac{dU_p}{dt} \equiv \left(\frac{\partial u}{\partial t}\right)_m + \left(\frac{\partial u}{\partial m}\right)_t \frac{dm}{dt} = \rho_0 U_s \left(\frac{\partial u}{\partial m}\right)_t , \tag{4.9}$$

and the Lagrangian equation for conservation of mass yields

$$\left(\frac{\partial V}{\partial t}\right)_m = \left(\frac{\partial u}{\partial m}\right)_t = \frac{1}{\rho_0 U_s} \frac{dU_p}{dt} , \tag{4.10}$$

for use in Equation (4.7), where the time derivative on the right in the above expression is evaluated from the pressure time derivative by

$$\frac{dU_p}{dt} = V_0 \frac{dP_H}{dt} / (S'U_p + U_s) . \tag{4.11}$$

The energy derivative in Equation (4.7) is obtained from the Lagrangian equation for conservation of energy,

$$\left(\frac{\partial E}{\partial t}\right)_m = - P \left(\frac{\partial V}{\partial t}\right)_m = - P_H \left(\frac{\partial V}{\partial t}\right)_m . \tag{4.12}$$

25

4.2   SENSITIVITY OF REACTION RATE TO EQUATION OF STATE

Since the Forest Fire reaction rates were obtained us-
ing the HOM equation of state (Reference 4) in Equation (4.1)
to evaluate the pressure for a partially reacted state, it
seemed worthwhile to recompute them using the HELP models in
order to check the sensitivity to variations in the explo-
sive equation of state. Such a calculation was done for
Composition B.

As a first step, the Fortran lisings given in Reference
1 were punched, and the program was run using the original HOM
equation of state in order to insure that there were no coding
errors. The reaction rates for Composition B were then re-
calculated using the equation of state models described in
Section III and the appropriate subroutines from the HELP
code. Comparison of Hugoniots for several unreacted mass
fractions is given in Figure 5, and a comparison of the
reaction rates as calculated from the two equations of state
is given in Figure 6. As the figure shows, the differences
of the equations of state are enough to very nearly account
for the factor of 1.5 necessary to bring the experimental
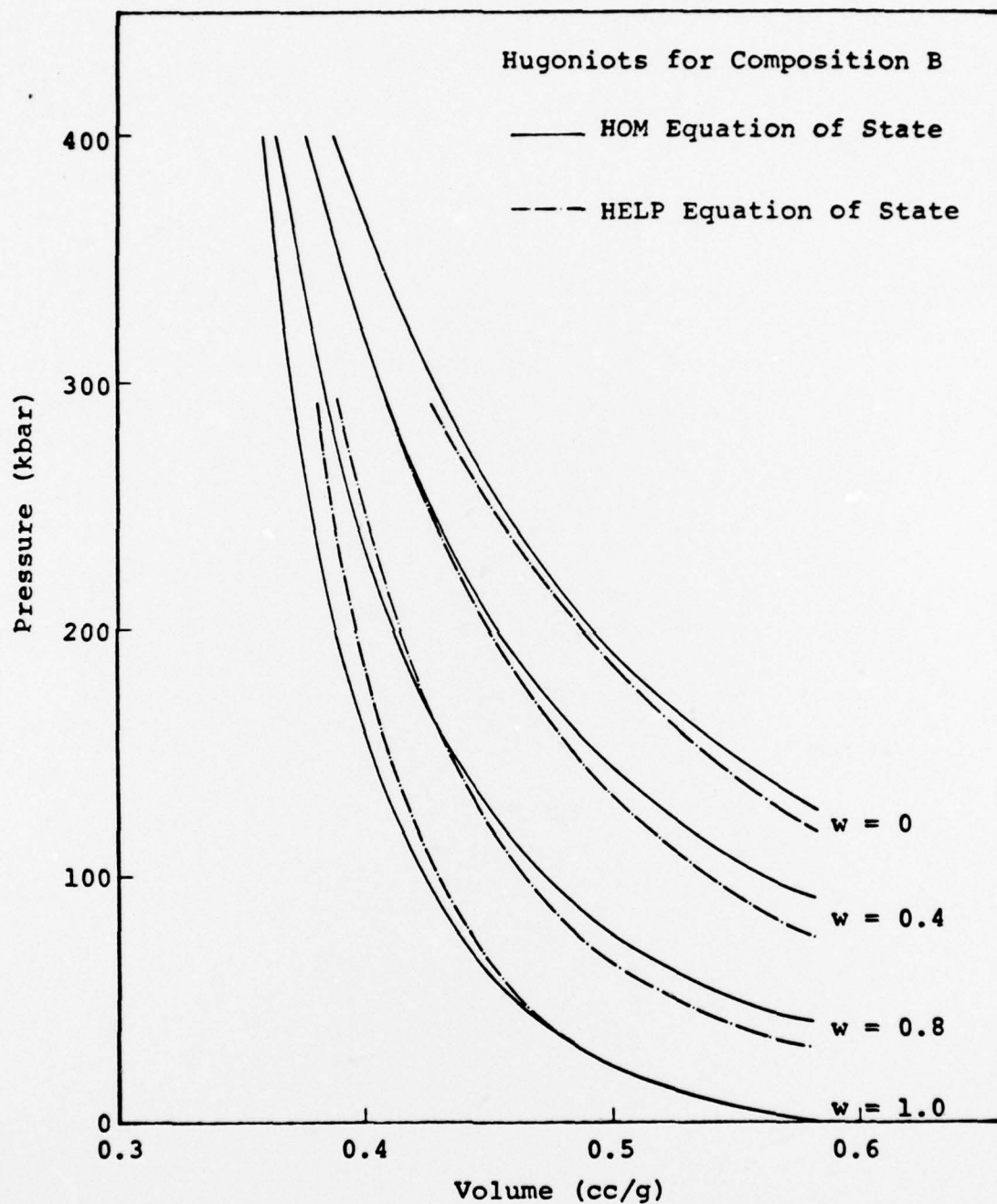and calculated detonation thresholds for Composition B into
agreement.

Figure 5. A comparison of several Hugoniots for Composition B as given by the HOM equation of state and the models used in the HELP code calculation, where w is the mass fraction of unreacted explosive.
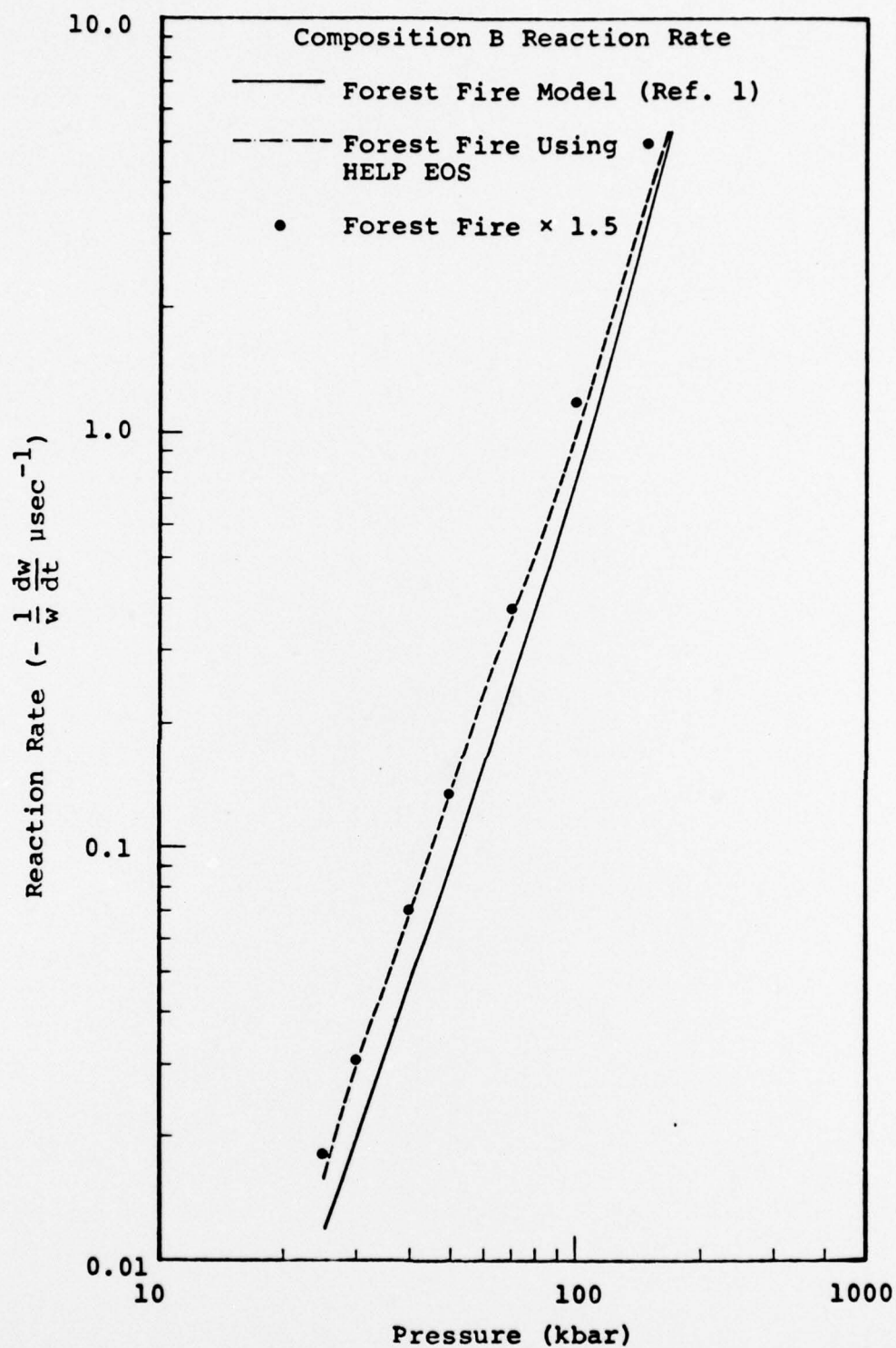
27

Figure 6. Comparison of Forest Fire reaction rates for Composition B as calculated with the HOM equation of state and the HELP equation of state models.

V. CONCLUSIONS

For the explosives PBX-9404 and Composition B the use
of the equation-of-state models presented here and the Forest
Fire reaction rates (modified for Composition B to account
for the sensitivity to the equation of state) results in good
agreement between calculated and observed velocity losses and
detonation thresholds for the penetration of unconfined explo-
sive slabs by steel spheres. The reaction-rate model (Reference
1), which was published after the start of the present contract,
has also been used with success at the Los Alamos Scientific
Laboratory to describe the detonation process in heterogeneous
explosives.

However, as pointed out in Reference 1, the use of this
reaction-rate model is not realistic at very low pressures.
For example, in the calculations reported here, the reaction
rates are assumed to be a function of pressure only, and they
are set to zero for pressures less than 15 kbar in PBX-9404
and 20 kbar in Composition B. Consequently, the model as
described here would not be useful for confined geometries
at low pressures, in which the explosive has a chance to
"cook", or for configuration in which the initial heating is
due principally to plastic deformation associated with macro-
scopic shear.

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